

Francisco R. Ortega - Alpha Shapes

▶ Motivation

- ▶ Research interest in Imaging and Computer Vision.
- ▶ Introduce myself to Computational Geometry

▶ Problem Statement

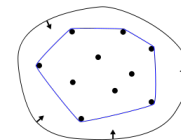
- ▶ Given a set of points, how best to described Shapes.
- ▶ Is there a way to find the corrected alpha value?

▶ Applications:

- ▶ Pattern Recognition, Shape Sample and Processing, Structural Molecular Biology

▶ Generalization of Convex Hull

- ▶ Think of a rubber band stretch around nails into a plane.
- ▶ For an alpha shape, when $\alpha = \infty$ then is a **convex hull**.



▶ Some Concepts for Alpha Shapes are:

- ▶ Alpha Complexes, K-Simplex, Delaunay Triangulations ,Voronoi Diagrams

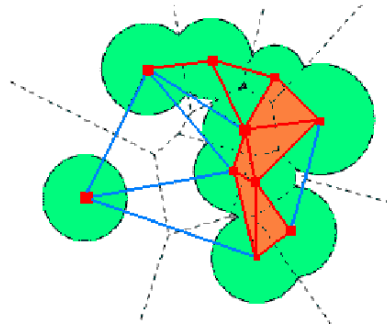
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- ▶ [Edelsbrunner and Mucke, 1994] ; Wikipidia

Alpha Shapes

- ▶ **Alpha Shapes are formed:**

- ▶ Set of balls with radius α . 0-ball is a point | ∞ -ball is an open-half space
- ▶ α -ball b is empty if $b \cap S = \emptyset$
- ▶ any subset $T \subseteq S$ for size $0 \leq k \leq 3$ defines k -simplex
- ▶ For $0 \leq k \leq 3$, k -simplex is α -exposed if α -ball is empty && $T = \partial b \cap S$
 - ▶ ∂b is the sphere or plane bounding b .
- ▶ The alpha shape of S is the polytope with boundaries of:
 - ▶ Triangles in F_2 , F_1 and vertices in F_0

Example

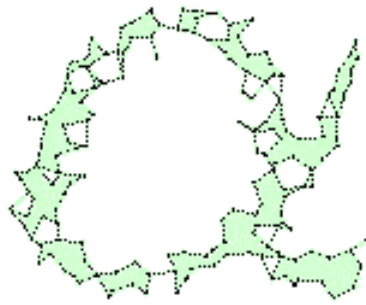


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- ▶ [Edelsbrunner and Mucke, 1994] ; <http://www.cs.tau.ac.il/~eitanyaf/>

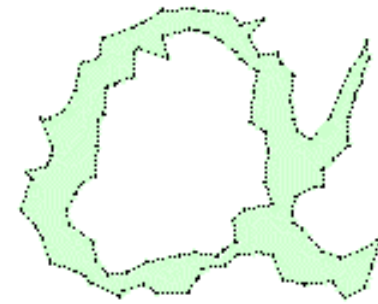
Sample Output



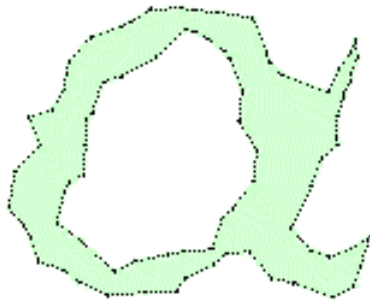
$$\alpha = k$$



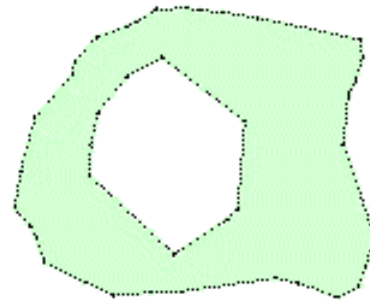
$$\alpha = 2k$$



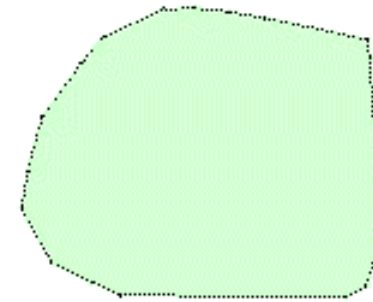
$$\alpha = 3k$$



$$\alpha = 4k$$



$$\alpha = 5k$$



$$\alpha = \infty$$

Objective / Deliverables

- ▶ **Cover required topics in CG for Alpha Shapes**
 - ▶ Convex Hulls, Voronoi Diagrams, Delaunay Triangulation
- ▶ **Emphasis in Alpha Shapes**
 - ▶ Theory
 - ▶ Applied Problems.
- ▶ **Provide visual demos and experiments.**
 - ▶ Matlab implementation using ASLIB.
- ▶ **References**
 - ▶ <http://www.cs.tau.ac.il/~eitanyaf/>
 - ▶ [Edelsbrunner and Mucke, 1994], [Edelsbrunner, 2008]
 - ▶ Computational Geometry, Berg et al.
 - ▶ Computational Geometry in C, O'Rourke

